



THE FIELD OF THE EXPERIMENT 70 ANALYSIS DIPOLES

David H. Saxon
Columbia University

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There are two dipoles of similar dimensions, having a length of 10 ft. steel and gaps of $7\frac{1}{4}$ " and $8\frac{1}{2}$ " by 10". The field is horizontal.

This note describes the mapping of the integral of the field along lines parallel to the axis of the system.

We take axes as shown in Figure 1. In the magnet gap, Maxwell's equations give

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad (1)$$

$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} \quad (2)$$

Define $\underline{C} = \int_{-\infty}^{\infty} B dz$ (3)

Then $\int \frac{\partial B_x}{\partial x} dz + \int \frac{\partial B_y}{\partial y} dz + B_z(z = \infty) - B_z(z = -\infty) = 0$ (4)

Setting $B_z(z = \pm\infty) = 0$

and interchanging the integral and differential operations
we obtain

$$\frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} = 0 \quad (5)$$

$$\frac{\partial C_x}{\partial y} = \frac{\partial C_y}{\partial x} \quad (6)$$

Hence we may set up a two-dimensional scalar potential
 Φ with

$$\underline{C} = -\text{grad } \Phi \quad (7)$$

The general solution is

$$\Phi = \sum_{n=0}^{\infty} (a_n r^n + b_n r^{-n-1}) e^{in\theta} \quad (8)$$

in polar co-ordinates where a_n and b_n are complex.

The boundary conditions are for $C_x(x,y)$ and $C_y(x,y)$

$$C_x(0,0) = \alpha \quad (\text{a constant}) \quad (9)$$

$$C_y(0,0) = 0 \quad (10)$$

Together with the approximate symmetries

$$C_x(x,y) \doteq C_x(x,-y) \quad (11)$$

$$C_x(x,y) \doteq C_x(-x,y) \quad (12)$$

Equation (12) is broken by the asymmetrical coil. Hence we can expand:

$$\begin{aligned}
 C_x(x,y) = & \alpha + \beta(x^2 - y^2) + \gamma(x^4 - 6x^2y^2 + y^4) \\
 & + \theta(x^6 - 15x^4y^2 + 15x^2y^4 - y^6) \\
 & + \tau(x^8 - 28x^6y^2 + 70x^4y^4 - 28x^2y^6 + y^8) \\
 & + \delta y + \epsilon(3x^2y - y^3) + \zeta x + \eta(x^3 - 3xy^2) \\
 & + \upsilon(x^5 - 10x^3y^2 + 5xy^4) \\
 & + \chi(x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6) \\
 & + \psi(28x^9 - 36x^7y^2 + 144x^5y^4 - 96x^3y^6 + 9xy^8) \\
 & + \dots
 \end{aligned} \tag{13}$$

$$\begin{aligned}
 C_y(x,y) = & -2\beta xy + 4\gamma(xy^3 - x^3y) + \delta x \\
 & - \theta(6x^5y - 20x^3y^3 + 6xy^5) \\
 & + \tau(8xy^7 - 56x^3y^5 + 56x^5y^3 - 8x^7y) \\
 & + \epsilon(x^3 - 3xy^2) - \zeta y + \eta(-3x^2y + y^3) \\
 & + \upsilon(-5x^4y + 10x^2y^3 - y^5) \\
 & + \chi(-7x^6y + 35x^4y^3 - 21x^2y^5 + y^7) \\
 & + \psi(-9x^8y + 96x^6y^3 - 144x^4y^5 + 36x^2y^7 - 28y^9) \\
 & + \dots
 \end{aligned} \tag{14}$$

The terms in α , β , γ , θ , τ have the required symmetries.

The field C_x was measured at various (x,y) locations with a flip coil 15' x $\frac{1}{2}$ " x 2 turns $\frac{1}{2}$ " apart. The charge from the flip coil is integrated to make a voltage proportional

$$D = \frac{1}{4w} \left\{ \int_{y-w}^{y+w} C_X(x+w, y') + C_X(x-w, y') dy' \right\} \quad (15)$$

where $w = 0.252''$ is half the flip coil width in x and y .

Hence $D = C_X + C_X'$, where to order w^2 (16)

$$\begin{aligned} C_{X'}(x, y) = & w^2 \left[\frac{2}{3} \beta + 4\gamma(x^2 + y^2 - 6xy) \right. \\ & + 10\theta(x^4 + y^4 - 6x^2y^2) \\ & + \tau \left[\frac{56}{3}(x^6 - y^6) + 280(x^2y^4 - x^4y^2) \right] \\ & + 2\epsilon y + 2\eta x + 20\nu \left(\frac{1}{3}x^3 - xy^2 \right) \\ & \left. + 14\chi(x^5 - 10x^3y^2 + 7xy^4) \right] + \dots \end{aligned} \quad (17)$$

This term proves to be 0.1% or less of C_X . It is somewhat smaller than the residuals on the fit but is included in the fit.

For Magnet No. 1 the field was mapped at three currents: 600.2 amps, 900.6 amps and 1100.0 amps, at 38 positions

$$x = -3'', -1.5'', 0'', 1.5'', 3''$$

at (18)

$$y = -4'', -3'', -1.5'', 0'', 1.5'', 3'', 4''$$

and

$$x = -1'', 0'', 1'' \quad \text{at} \quad y = 5''$$

The limits of the aperture are $x = \pm 3.625''$
 $y = \pm 5''$.

The field measurements were divided by $C_x(0,0)$ and fitted to the polynomial (16) to minimize the sum of squares of differences. Different orders of polynomial were tried to optimise the fit with the least number of parameters. The constants β , γ , etc., are permitted a linear dependence on $C_x(0,0)$, as follows

$$\text{Define } R_1 \equiv \frac{C_x(0,0)}{C_x(0,0)_{I = 900.6 \text{ A}}} - 1 \quad (19)$$

The best fit is shown in Table I.

The RMS deviations in parts per thousand are

I = 600 A	RMS	1.84	38 pts	
900		1.63	38	(20)
1100		1.69	38	
Total		1.72	114 readings.	

The point by pt values and residuals are given in Table II, in the format

(x,y)
C₆₀₀ δ_{600}
C₉₀₀ δ_{900}
C₁₁₀₀ δ_{1100}

where C_I is the measured value at I amps
and

δ_I is the difference with the fit.

$$\delta_I = (C_I^{th} + C_{I'}^{th}) - C_I^{exp} \quad (21)$$

The agreement is considered adequate.

It is also necessary to measure the dependence of $C_X(x,y)$ on I, the current. For this absolute and not relative numbers are required.

Now

$$V = - \frac{\partial}{\partial \tau} \int \underline{B} \cdot d\underline{S} \quad (22)$$

so

$$V_\tau = 2nw \int B_X dz \quad (23)$$

$$= 2nwC_X \quad (24)$$

where

n = no. of coil turns = 2

w = coil width = 0.5040 in.

= 1.278 cm

τ = time constant

$$\tau = .2978 \left(1 + \frac{R_{flip \ coil}}{R_{integrator}} \right) \text{ sec.} \quad (25)$$

$R_{flip \ coil} = 150.0 \ \Omega$

$$R_{\text{integrator}} = 301 \text{ k}\Omega$$

so

$$\tau = .2980 \text{ sec.} \quad (26)$$

Hence

$$C_x = 5.831 \text{ V weber/m.} \quad (27)$$

This is checked by calculating the effective length

$$C_x(0,0) \equiv B_x(0,0,z=0) \ell_{\text{eff}} [\text{defines } \ell_{\text{eff}}] \quad (28)$$

and at 1100 amps

$$B_x(0,0,0) = 1.254 \text{ Tesla} \quad (29)$$

(measured with Hall probe)

$$V = -.6965 \text{ volts} \quad (30)$$

Hence

$$\ell_{\text{eff}} = 3.34 \text{ m} = 10'7.2". \quad (31)$$

Compare the nominal effective length

$$\begin{aligned} \ell_{\text{eff}} &= \text{length} + \text{width} \\ &= 10'7.25" \end{aligned} \quad (32)$$

Measurements of voltage were made at every 100 amps from 0 to 1100 amps and fitted to the form

$$C_x(0,0) = \sum_{n=1}^4 A_n I^n \quad (33)$$

where I is measured in kiloamps.

The results are shown in Table III. Different orders of polynomial were tried and the results in Table III represent

the preferred fit, with

$$\begin{array}{ll} A_1 & 3.8216 \\ A_2 & .25645 \\ A_3 & .42835 \\ A_4 & .27875 \end{array} \quad (34)$$

To compute $C_X(x,y,I)$, we use the form

$$C_X(x,y,I) = \sum_{n=1}^4 A_n I^n C_X(x,y) \quad \text{weber/m} \quad (35)$$

using the series (34) for A_n

I in kiloamps

and the series (13) for $C_X(x,y)$.

The results on Magnet No. 2 are as follows:

The field was mapped at 600.2, 900.0, 1100.6 amps at 37 positions.

$$x = -3.6'', -1.8'', 0'', 1.8'', 3.6''$$

at

$$y = -4'', -3'', -1.5'', 0'', 1.5'', 3'', 4'' \quad (36)$$

and

$$x = -1'', 0'' \text{ at } y = 5''.$$

The limits of the aperture are

$$x = \pm 4.25''$$

$$y = \pm 5''$$

as for Magnet No. 1. Define

$$R_2 \equiv \frac{C_x(0,0)}{C_x(0,0)_I = 900.0A} - 1 \quad (37)$$

The best fit is shown in Table IV.

The RMS deviations in parts per thousand are

I = 600 A	RMS	1.11	37 pts
900		1.22	37
1100		1.24	37
Total		1.19	111 readings.

(38)

The point by point values and residuals are given in Table V in the format of Table II. The agreement is considered adequate.

The value of $\int B d\ell$ at $x = y = 0$, $C_x(0,0)$ was measured as a function of current and fitted to the form (33). The results are shown in Table VI.

The preferred fit has

$$\begin{aligned} A_1 & 3.1821 \\ A_2 & 0.10158 \\ A_3 & -0.089806 \\ A_4 & 0 \end{aligned} \tag{39}$$

Measurements were made on Magnet No. 2 of fringe field and reproducibility, and on Magnet No. 1 of hysteresis. The results are considered adequate.

The fringe field is ~2.0% at points 2' from the magnet steel end; negligible at 4'.

Field values are reproducible within 0.2% after turning off and on again, or after turning to reversed field and on again.

Allow 5 to 10 minutes for field to stabilise after switching to a new current.

The maximum permissible fields are governed by considerations of cooling and available voltage.

Let Q gal/min. be the water flow and P the available pressure in psi. Measurements were made with about 100' of 1" diameter hose in series and two quick-disconnects. The data are well-described by the form

$$Q^2 = k(P - P_0) \tag{40}$$

We find

	<u>Magnet 1</u>	<u>Magnet 2</u>	
k	4.80	4.44	
P _o	7	9	(41)

We have specified a requirement of $Q = 20$ for each magnet.

Hence

$$\begin{array}{ll} P = 90 & \text{Magnet 1} \\ & \text{99 Magnet 2} \end{array} \quad (42)$$

The available pressure is 120 - return value, so the pressure is probably adequate.

NOTE: Water enters in manifold at top centre of magnet and leaves through manifold at top side. There is a temperature interlock circuit, normally closed, in which "Woods-metal" buttons melt at 194°F. Extra buttons are available from R. McCracken or Belmont Smelting and Refining Company, Brooklyn, New York.

We chose:

Maximum operating temperature: 174°F.

John Peoples predicts a maximum inlet temperature of 113°F at 95°F ambient temperature.

Then 20 gal/min. carry away: 173 kW

With an inlet temperature of 50°F,

20 gal/min. carry away: 346 kW

Coil resistance at 174°F is 0.123Ω.

Hence maximum permissible current is

1183 amps at 113°F water inlet

1670 maps at 50°F water inlet.

A maximum of 1183 amps was conceived in the design. This gives

$$\begin{aligned} \int B dl &= 4.33 \text{ Tesla m Magnet 1} \\ &3.70 \text{ Magnet 2} \end{aligned} \quad (43)$$

The two magnets are supplied in series by a single Transrex supply. This gives a maximum current of

1250 amps at 400 volts.

α	=	1	
β	=	.0016637	+ .00045073 R_1
γ	=	-.000078425	+ .0000058173 R_1
δ	=	.00026953	+ .000023640 R_1
ϵ	=	-.000028744	- .000005290 R_1
ζ	=	-.0030665	- .00063054 R_1
η	=	0	
θ	=	.0000019521	+ .0000001002 R_1
τ	=	.0000000745	+ .0000000125 R_1
υ	=	0	
χ	=	0	
ψ	=	0	

TABLE I.

Coefficients of Fit to Field Shape

Magnet No. 1

TABLE II.

Magnet 1 Field Shape

		(-1, 5)	(0, 5)	(1, 5)	
		915 2	914 -1	922 1	
		912 3	912 0	919 2	
		910 2	908 -1	916 1	
(-3, 4)	(-1½, 4)	(0, 4)	(1½, 4)	(3, 4)	
1053 0	970 -1	955 0	981 -2	1079 6	
1054 0	968 -1	953 0	979 -1	1078 2	
1054 0	966 -2	951 -1	975 -2	1076 1	
(-3, 3)	(-1½, 3)	(0, 3)	(1½, 3)	(3, 3)	
1019 -1	990 0	980 0	999 -1	1040 -1	
1021 0	989 0	979 0	999 -1	1040 -2	
1023 0	988 0	978 -1	997 -1	1041 -2	
(-3, 1½)	(-1½, 1½)	(0, 1½)	(1½, 1½)	(3, 1½)	
1003 5	998 1	997 0	1007 0	1020 2	
1004 4	998 1	996 0	1007 0	1021 2	
1005 5	997 1	996 0	1006 0	1022 2	
(-3, 0)	(-1½, 0)	(0, 0)	(1½, 0)	(3, 0)	
1000 0	999 1	1000 0	1008 0	1016 -3	
1000 -1	999 0	1000 0	1008 0	1017 -3	
1002 -1	999 0	1000 0	1007 -1	1017 -3	
(-3, -1½)	(-1½, -1½)	(0, -1½)	(1½, -1½)	(3, -1½)	
1003 3	997 0	996 0	1006 0	1020 1	
1004 2	997 0	995 0	1006 0	1021 2	
1005 2	997 0	995 0	1006 0	1022 2	
(-3, -3)	(-1½, -3)	(0, -3)	(1½, -3)	(3, -3)	
1021 -2	988 -1	978 0	997 0	1039 -3	
1022 -3	988 0	977 0	996 0	1041 -1	
1025 -2	987 -1	976 0	995 0	1041 0	
(-3, -4)	(-1½, -4)	(0, -4)	(1½, -4)	(3, -4)	
1053 -3	967 -1	951 1	977 0	1077 2	
1057 0	965 -1	949 2	974 0	1078 5	
1055 -2	963 -2	947 2	971 -1	1077 4	

Magnet No. 1. Central Field.

kAmps	Volts	C_x Measurement	Residual
0	.0003	.0018	-.0018
.1002	.0653	.3814	-.0036
.2003	.1299	.7586	-.0023
.3003	.1944	1.1353	-.0004
.4004	.2589	1.5120	.0013
.5007	.3235	1.8892	.0018
.6006	.3875	2.2630	.0020
.7009	.4514	2.6362	.0011
.8000	.5140	3.0018	-.0006
.9003	.5766	3.3673	-.0027
.9993	.6371	3.7207	-.0029
1.1001	.6965	4.0676	.0031

TABLE III.

α	=	1	
β	=	.0014324	+ .00013822 R_2
γ	=	-.000047086	- .0000000492 R_2
δ	=	.00026473	- .000090142 R_2
ϵ	=	0	
ζ	=	.0028478	- .00033890 R_2
η	=	0	
θ	=	.00000025762	- .000000081390 R_2
τ	=	.000000037910	+ .0000000025749 R_2
υ	=	0	
χ	=	0	
ψ	=	0	

TABLE IV.

Coefficients of Fit to Field Shape

Magnet No. 2

TABLE V.

Magnet 2 Field Shape

		(-1,5)		(0,5)					
		943	3	945	-2				
		943	3	944	-2				
		942	4	944	-2				
(-3.6,4)		(-1.8,4)		(0,4)		(1.8,4)		(3.6,4)	
1051	1	976	0	968	0	987	-1	1073	0
1052	1	976	0	967	0	985	-2	1072	-1
1053	3	975	0	967	0	985	-1	1072	-1
(-3.6,3)		(-1.8,3)		(0,3)		(1.8,3)		(3.6,3)	
1020	0	991	0	985	0	1002	-1	1042	-1
1021	-1	991	0	984	0	1001	-1	1042	-1
1022	0	991	0	984	0	1001	-1	1042	-1
(-3.6,1.5)		(-1.8,1.5)		(0,1.5)		(1.8,1.5)		(3.6,1.5)	
1004	3	998	0	997	0	1008	0	1025	1
1004	2	998	0	997	0	1008	0	1025	1
1005	2	998	0	997	0	1008	0	1025	2
(-3.6,0)		(-1.8,0)		(0,0)		(1.8,0)		(3.6,0)	
1000	-1	998	0	1000	0	1009	0	1020	-2
1000	-2	999	0	1000	0	1009	0	1021	-2
1001	-2	999	0	1000	0	1009	0	1020	-2
(-3.6,-1.5)		(-1.8,-1.5)		(0,-1.5)		(1.8,-1.5)		(3.6,-1.5)	
1003	1	997	0	996	0	1008	0	1024	2
1003	1	997	0	996	0	1008	0	1024	2
1004	1	997	0	996	0	1007	0	1024	2
(-3.6,-3)		(-1.8,-3)		(0,-3)		(1.8,-3)		(3.6,-3)	
1018	-2	990	-1	983	0	1000	0	1040	0
1019	-2	989	-1	983	0	1000	0	1041	1
1020	-2	990	-1	983	0	999	0	1041	1
(-3.6,-4)		(-1.8,-4)		(0,-4)		(1.8,-4)		(3.6,-4)	
1049	0	975	-1	966	1	985	0	1070	2
1049	-1	974	-1	966	1	984	0	1070	2
1050	-1	974	-1	965	1	982	0	1070	2

Magnet No. 2. Central Field.

kAmps	Volts	C _x Measurement	Residual
0	.0003	.0017	-.0017
.1003	.0546	.3181	.0020
.2003	.1098	.6402	.0005
.3004	.1653	.9639	-.0012
.3996	.2199	1.2822	-.0002
.5007	.2757	1.6076	-.0001
.6010	.3310	1.9306	-.0010
.7002	.3851	2.2455	.0016
.8004	.4401	2.5662	-.0002
.8999	.4939	2.8799	.0005
.9999	.5478	3.1942	-.0006
1.0999	.6008	3.5033	.0001

TABLE VI.

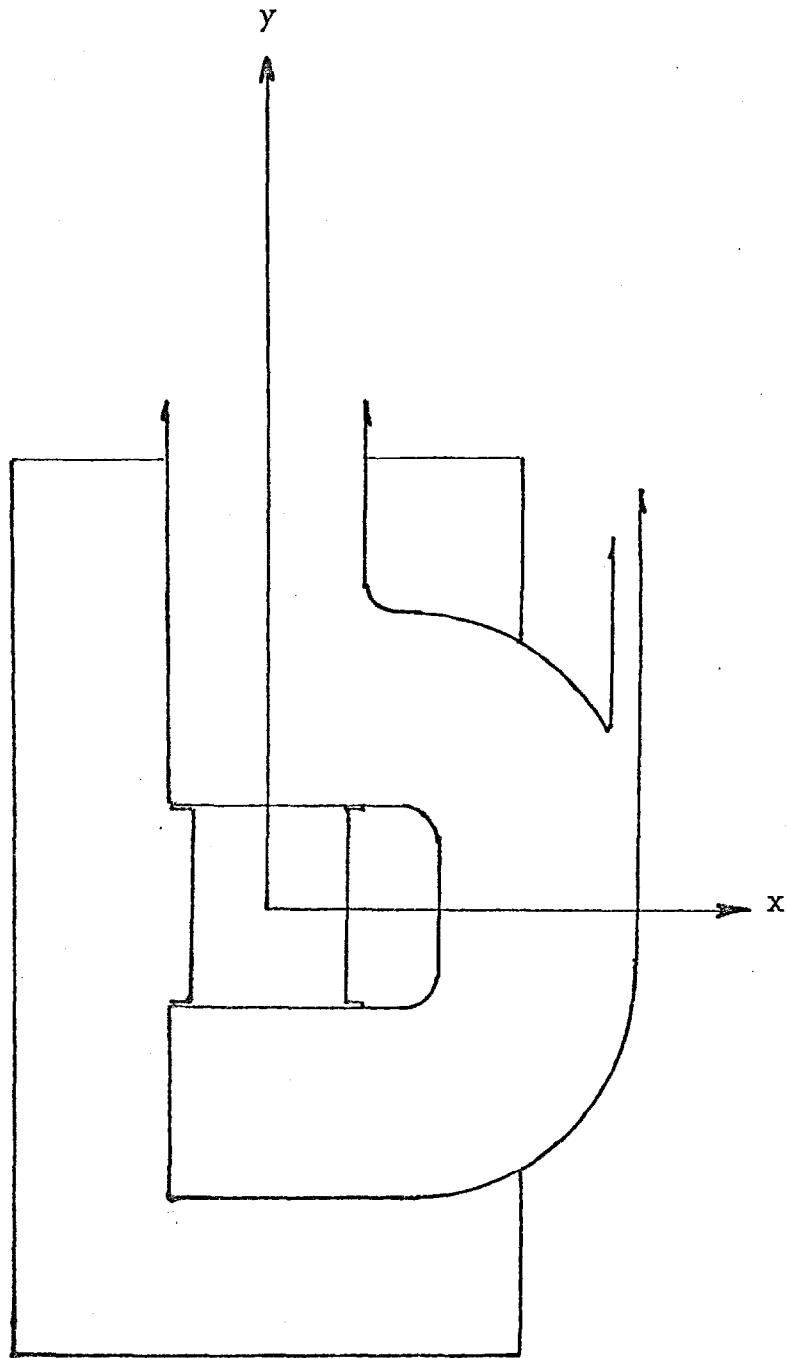


Figure 1.

Magnetic Field Axes